3: reason and argument

Probability review problems

Let's say we have a test for some disease that is 95% selective. You do NOT have the disease. How many times would you have to take the test in order to make it more likely than not that you will return at least one positive result despite not having the disease?

Let's say 10% of the population can safely donate blood to you. We have a test for blood type that is 90% selective and 90% sensitive. Someone tests positive for being a donor. What is the probability that she is a donor?

Two people named 'Lauren', Lauren A and Lauren B, each order three burritos at Boloco. Lauren A orders three Tofu Teriyaki while Lauren B orders two Tofu Teriyaki and one Beef Bangkok. "Lauren!" they yell, showing two bags, with three burritos each. Lauren A takes a TT burrito out of a bag and proclaims: "This one is probably mine." Lauren B objects: "You can't know that. I ordered TT burritos too."

(a) Is Lauren A right, i.e., is Pr(The bag is Lauren A's bag|she found a TT burrito in it) > ½? What is the specific value?

(b) Lauren A then pulls another TT burrito from the bag. What is Pr(Bag A is Lauren A's bag |She pulls two TT burritos from it)?

Monty Hall has a new game. It's just like the old Make-a-deal scenario: 3 doors, two goats, one Maserati. You choose a door, say door A. Here's the new part: he then flips a coin. If it comes up heads, he opens door B, if it comes up tails he opens door C. Let's say it comes up heads and he opens door B, revealing a goat. He then gives you the chance to switch your choice from A to C. Is it to your advantage to switch? Explain.

It rains a lot in the Pacific Northwest, especially in the winter: 2 out of 3 days in winter are rainy, while just one is sunny. In the summer, 2 out of 3 days are sunny and just one is rainy. You wake up from a coma in Twin Peaks, Washington. You know that it's either winter or summer, and you can assume Pr(winter) = ½ = Pr(summer). You noticed that the past three days have been sunny, what is the probability that it is winter? Show your work and clearly indicate your answer.

Let's say 5% of nickels are unfair, in that when flipped they come up heads 60% of the time. The other 95% of nickels are fair. You flip a nickel 10 times, and every flip is heads. What is the probability that your nickel is unfair?

Half the time, your gas gauge is accurate, but half the time, it just says you have ½ a tank of gas, regardless of whether that's true. The Dude has just returned your car. 70% of the time, he returns it with an almost empty tank, while 30% of the time, he returns it half full. You get in your car and see the gauge indicating ½ tank. In these circumstances, what is the probability that the tank is half full? Show your work and clearly indicate your answer.

The gas gauge on your new car always works well. One of your friends borrowed it last night without asking, but you're not sure whether it was The Dude or Maude. 70% of the time, it was the Dude, while 30% of the time, it was Maude. As you know, 70% of the time, The Dude returns your car with an almost empty tank, while 30% of the time, he returns it half full. Maude, more virtuous, returns it half full 70% of the time and empty only 30% of the time. You get in your car and see the gauge indicating ½ tank. In these circumstances, what is the probability that Maude borrowed the car? Show your work and clearly indicate your answer.

Let's say we have two bags, each of which contains a bunch of Scrabble tiles (you know: little wooden things, each of which has a single letter on it). The tiles in one bag spell out 'battleship', while the tiles in the other spell out 'bayonet'. You are looking for the 'bayonet' bag. If you choose a bag at random, the probability that you get the 'bayonet' bag is obviously ½. (Tricky!)

What is the probability that you have the 'bayonet' bag if you draw a 'b' from it?

What is the probability that you have the 'bayonet' bag if you draw a 't' from it?

What is the probability that you have the bayonet bag if you draw a 't' from each bag?

Jack and Jill pack their lunch for a hike up the hill. Jack's bag of sandwiches: two cheese, one tuna. Jill's: two tuna, one cheese. They sit down and each pulls one bag from the backpack, but they're not yet sure whose is whose. They reach in to their respective bags and withdraw one sandwich each. What is the probability that they have the correct bags (i.e. Jack has Jack's bag and Jill has Jill's) if…

…Jack draws a tuna and Jill draws a cheese?

…Jack draws a tuna and Jill draws a tuna?

…Jack draws a cheese and Jill draws a tuna?

…Jack draws a cheese and Jill draws a cheese?

We want to know who has the disease. We have two tests, described below. Let's use 'd' to represent the claim that someone has the disease, and 't' to represent the claim that someone tests positive for the disease.

The base rate of the disease is 1%:		pr(d) = 1/100
Test 1:	Sensitivity 70%:	pr(t | d) = 7/10
	Selectivity 100%:	pr(t | -d) = 0
Test 2:	Sensitivity 100%:	pr(t | d) = 1
	Selectivity 70%:	pr(t | -d) = 3/10
	Note: 70% selectivity means 30% false positive rate.	

For each test, calculate pr(d | t).

Consider the following testing strategies:

A: First test everyone with Test 1. Then apply Test 2 only to those who tested positive to Test 1.

B: First test everyone with Test 2. Then apply Test 1 only to those who tested positive to Test 2.

C: First test everyone with Test 2. Then apply Test 2 again only to those who tested positive the first time.

For each strategy, calculate the probability that a person has the disease, given that she has tested positive in both rounds of testing.

For which strategy does the greatest number of sick people test positive in both rounds?

If it were essential to isolate all and only people with the disease, how could you do that with repeated applications of these two tests? Explain.

The philosophy and religion departments are gearing up for their annual holiday raffles. Each raffle has 100 tickets and they both always sell out. For each raffle, the tickets cost the same, and the prizes are the same. Each raffle has exactly one winner and the winner is the one who buys the winning ticket. You can afford to buy four tickets. Which strategy, if any maximizes the probability that you win at least one raffle?

1. Buy two tickets in each raffle.
2. Buy four tickets in one raffle.
3. Buy 3 in one and 1 in the other.

Or is it true that

4. All of the previous strategies are equally likely to produce a win.
5. The first two are equally likely to produce a win.

What is the probability that:

you win the philosophy raffle if you buy four tickets?

you win the philosophy raffle if you buy two tickets?

you win the philosophy raffle if you buy one ticket?

you win philosophy or religion if you buy two tickets in each?

you win philosophy or religion if you buy one phil and 3 rel?

you win philosophy *and* religion if you buy two in each?

you win philosophy and religion if you buy one phil and 3 rel?

you win philosophy and religion if you only buy 4 phil tickets?

Which of the three strategies above (1, 2, or 3) has the greatest expected monetary value?

Game night in the Philosophy Department was cancelled a couple of years ago, after a professor, out of money, tried to bet his dog on a game of Woozy. It's a very fun game. Without looking, you pick two cards from a deck. If they are consecutive, you win, if not, you lose, and everybody yells "Woozy!". It costs $40 to play, but you are given $250 if you win. Just in case you don't know, a standard deck includes 52 cards: four suits—hearts, diamonds, clubs, and spades—each of which includes 13 cards—A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K. Consecutive cards are next to one another in order, so, 4 and 5, or 4 and 3 are consecutive. King and Ace or King and Queen are consecutive. Ace and 2, or 2 and 3 are consecutive, and so on. Suits don't matter for consecutiveness, so Ace of spades and King of hearts are consecutive. Identical cards—4 and 4, e.g.—are not consecutive.

What it the probability of winning Woozy?

What is the expected monetary value of playing Woozy?

Is it a good idea, financially speaking, to play Woozy? Explain.

If and only if you win, you are offered the chance to "go WoozyWoozy". You bet $250, keep the first two cards you drew, and draw a third card from the deck. If it is consecutive with the other two you win $1500. Otherwise, you lose.

Assume you have already won Woozy. What is the probability of winning WoozyWoozy?

What is the expected monetary value of playing WoozyWoozy?

Is it a good idea, financially speaking, to go WoozyWoozy? Explain.

Would you go WoozyWoozy?