

# reason and argument

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## Answers to validity and formalization questions

For each of the following examples, provide a schematization that captures how the truth of the whole statement depends on the truth of its parts, if there are any.

*For these answers, I try to make it obvious which letter refers to which statement part. You should be explicit about which sentence letter stands for which part of the whole statement.*

You can't park there unless you're a cop, but I can valet it for you.

$(\neg p \vee c) \& w$  or  $(p \rightarrow c) \& w$

*There are many accurate schematizations that "capture how the truth of the whole statement depends on the truth of its parts." You could schematize this accurately as 'p', but that would not capture what I asked you to capture. The same holds for all of the following statements.*

Without trying to make excuses, he admitted to the crime and asked for forgiveness.

$(\neg e \& a) \& f$

*You can group any two of these within parentheses.*

He pulled through despite doctors' dire predictions.

$p \& d$

*Words like 'despite' do more than merely connect statements. They comment on the connection. Truth-functionally, however, it works just like 'and', and you can see this by considering when you would say someone who used this phrase had said something false. For example, if the doctors had made no dire predictions, the statement would be false, just as it would if he hadn't pulled through.*

He said he's rested and ready. (Think of two options here.)

$p$  or  $p \& q$

*That is, this might not be a compound statement. Why? Well, this is a piece of indirect discourse and you might have qualms about breaking up the part where one explains what someone else said. Compare: He said "I'm rested and ready."*

Though ready, he is neither fit nor trim.

$r \& \neg(f \vee t)$  or  $r \& (\neg f \& \neg t)$

*Can you show that these are logically equivalent expressions?*

I never thought about love when I thought about home.

$h \rightarrow \neg l$

*I thought about home only if I didn't think about love.*

You want to know how to rhyme you better learn how to add.

$r \rightarrow a$  or  $\neg r \vee a$

*You can't learn how to rhyme unless you learn how to add.*

If the federal government shuts down on Tuesday, some employees will continue reporting to their departments and agencies, while others will be furloughed. (*NYT*, 9/29/13)

*Shutdown*  $\rightarrow$  (*Report*  $\&$  *Furloughed*)

$S \rightarrow (R \& F)$

You rush a miracle man, you get rotten miracles.

- Billy Crystal in *The Princess Bride* (1987)

*Rush*  $\rightarrow$  *Rotten Miracles*

$R \rightarrow M$

Without a complete capitulation by house Republicans, large sections of the government would close, hundreds of thousands of workers would be furloughed without pay, and millions more would be asked to work for no pay. (*NYT*, 9/29/13)

*Capitulation*  $\vee$  (*Close*  $\&$  *Furlough*  $\&$  *No Pay*)

$C \vee ((G \& F) \& N)$ ;  $\neg C \rightarrow ((G \& F) \& N)$

The House's unruly conservatives had more than enough votes to defeat a spending bill that would not do significant damage to the health care law, unless Democrats were willing to bail out the speaker. (*NYT*, 9/29/13)

*Defeat*  $\vee$  *Bail*  $D \vee B$

Mr. McGovern would love to buy health insurance, but only if it costs him less than \$100 a month. (*NYT* altered, 9/29/13)

$(M \rightarrow C) \& (C \rightarrow M)$  "but only if" suggests a biconditional

It is not the case that either I always drink beer, or when I do I don't drink *Dos Equis*.

$-(\text{Always } v (D \rightarrow \text{-DosEquis})) \quad \text{-}(A \vee (D \rightarrow E))$   
*Don't collapse (D → E) into one sentence letter!*

Efforts to teach the language to children on the reservation have grown, **with** more schools offering classes, **but** most people who count Navajo as their first language are now elderly. (NYT 9/26)

$(T \ \& \ C) \ \& \ F$

**Although** today's new moon takes place in the most sensitive area of your chart there is nothing you should be too worried about. (Scorpio Horoscope, NY Post, 9/24/14)

$M \ \& \ W$

**If, and only if**, you are feeling ambitious, you could make pizza dough tonight. (NYT, 9/10/14)

$(A \rightarrow P) \ \& \ (P \rightarrow A)$

**Unless** the Lord had been my help, my soul had almost dwelt in silence. (Psalms 94:17, KJV)

$H \vee S$

Marry, **and** you will regret it; do not marry, **and** you will regret it; **but** churros are a must at a bridal shower.

(KimKierkegaardashian, Twitter, 9/1/14)

$[(M \rightarrow R) \ \& \ (\text{-}M \rightarrow R)] \ \& \ C$

*Notice here that 'and' is used to indicate a conditional!*

Cersei seemed quite interested in saving him, **but only if** he isn't weakened by the treatment. (NYT, 6/16/14, seriously)

$(S \rightarrow T) \ \& \ (T \rightarrow S)$

We will not sign any deal unless all sanctions are lifted on the same day. – Hassan Rouhani quoted *Reuters* 4/9/15

$\text{-}d \vee l, d \rightarrow l$

*d: we will sign a deal; l: all sanctions are lifted same day*

Merkel warns that Greece will receive aid only if reforms are met.  
– *NYT* 3/20/15

*m: M warns that G will get aid only if reforms are met.  
There is a conditional in the sentence, but it is within the  
scope of what Merkel warns about, so we can't unpack it as  
a truth-functional part of the whole.*

Japanese Prime Minister Shinzo Abe should only be invited to  
address the U.S. Congress during an expected visit if he  
acknowledges Japan's World War Two past. – *Reuters* 3/19/15

*i → a  
i: Abe should be invited; a: Abe acknowledges past*

Strive not with a man without cause, if he have done thee no  
harm. – Job 3:30, *KJV*

*n → s  
n: he have done thee no harm; s: strive not with a man*

*n → ~s  
n: he have done thee no harm; s: strive with a man*

If they were intending to laugh at me there would be nothing  
unpleasant in their spending their time in court laughing and  
jesting, but if they are going to be serious, the outcome is not  
clear except to your prophets. – Plato, *Euthyphro* Grube, trans.

*This one is a big ugly mess. The easiest way to go:*

*l: they intend to laugh at me  
n: there is nothing unpleasant in their spending their time in court  
laughing and jesting*

*s: they are going to be serious  
c: the outcome is not clear except to your prophets*

*(l → n) & (s → c)*

*Another way to go:*

*l: they intend to laugh at me*

*n: there is nothing unpleasant in their spending their time in court laughing*

*m: there is nothing unpleasant in their spending their time in court jesting (continued...)*

*s: they are going to be serious*

*c: the outcome is not clear to anyone except to your prophets*

*d: the outcome is clear to your prophets*

$(l \rightarrow (m \ \& \ n)) \ \& \ (s \rightarrow (c \ \& \ d))$

*And one more:*

*l: they intend to laugh at me*

*n: there is nothing unpleasant in their spending their time in court laughing*

*m: there is nothing unpleasant in their spending their time in court jesting*

*s: they are going to be serious*

*c: the outcome is not clear*

*d: you are a one of their prophets*

$(l \rightarrow (m \ \& \ n)) \ \& \ (s \rightarrow (c \ \vee \ d))$  or, equivalently:

$(l \rightarrow (m \ \& \ n)) \ \& \ (s \rightarrow (-c \ \rightarrow \ d))$

*Notice that different answers require thinking differently about the statements that constitute the whole. The clearest feature of the claim is that it is a conjunction, each part of which is a conditional.*

Indicate whether each of the following arguments is valid (V/I):

\_V\_ If something is red, then everything is red. Everything is red. Therefore, something is red.

\_V\_ Some dogs are bigger than other dogs. Therefore, some dogs are smaller than other dogs.

*Notice: it's hard to make this argument look formally valid, if all you have to work with is sentential logic.*

\_V\_ If that's a Rolex, then I'm the Pope. I'm not the Pope. So, that's not a Rolex.

\_I\_ You can drive only if you have a license. You have a license. So, you can drive.

\_I\_ Only red cards win the game. This is a red card. So this card wins the game.

*Notice: it's hard to make this argument look formally valid, if all you have to work with is sentential logic. People also find this one hard to get right. Many think it's valid.*

\_I\_ You can drink here only if you are 21 or over. You are 22. Therefore, you can drink here.

\_I\_ The United States will act only if surrounded by a broad coalition. (NYT, 9/24/14, altered)

The US is surrounded by a broad coalition.

Therefore, the US will act.

*There might be other conditions we need to meet before the US acts.*

\_\_V\_\_ Half the options being granted to senior executives this year will accrue to them only if the company hits its earnings targets in 2016. (NYT, 7/24/14)

All the options being granted to senior executives this year will accrue to them.

Therefore, the company hits its earnings targets in 2016.

*If all the options are granted, then half are, which is enough to establish that the conclusion is true.*

Indicate whether each of the following statements is true ('T') or false ('F').

- T All sound arguments are valid.
- F All valid arguments are sound.
- F An argument is sound if it is valid.
- T An argument is sound only if it is valid.
- F An argument is sound, unless it is valid.
- F Only sound arguments are valid.
- F A valid argument has at least one true premise.

Fill out the truth table and determine whether or not the following argument form is valid.

$\sim p \ \& \ q$                       Circle one:    **valid**    invalid

$\sim(q \rightarrow p)$

$\sim p$

*Notice there are no parentheses around the first premise: it's not a negated conjunction. And notice that the premises are logically identical.*

	Premise	Premise	Conclusion	Whole argument in one schema
p	Q	$\sim p \ \& \ q$	$\sim(q \rightarrow p)$	$[(\sim p \ \& \ q) \ \& \ \sim(q \rightarrow p)] \rightarrow \sim p$
T	T	F	F	T
T	F	F	F	T
F	T	T	T	T
F	F	F	F	T

*Schematize this argument in a manner that shows how the truth of each statement depends on the truth of its parts, if any.*

She doesn't weigh the same as a duck unless she's made of wood.

She's made of wood only if she's a witch.

She weighs the same as a duck.

Therefore, she's a witch.

$\neg D \vee M$  (which is just:  $D \rightarrow M$ )

$M \rightarrow W$

D

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W

*Write out a truth table that includes columns for each of the premises and the conclusion of your schematized argument.*

D	M	W	$\neg D \vee M$	$M \rightarrow W$	D	W
T	T	T	T	T	T	T
T	F	T	F	T	T	T
T	T	F	T	F	T	F
T	F	F	F	T	T	F
F	T	T	T	T	F	T
F	F	T	T	T	F	T
F	T	F	T	F	F	F
F	F	F	T	T	F	F

*Does your schematization show that the argument is valid, or not?*

*(2 points)*

**Yes.**

*Explain your answer.*

*(3 points)*

**Every row in which all of the premises are true is also a row in which the conclusion is true.**

There are many ways to schematize any sentence accurately, even if not all ways of doing so are particularly useful. For each of the following propositional forms, indicate whether it is an accurate schematization of the sentence below, for some assignment of letters to parts of the claim. Use 'Y' for yes and 'N' for no.

Physics is not flawed, but I can't have free will unless physics is flawed.

Ask yourself: what is the main connector? What are the secondary connectors? Can I convert one into the other?

Y N  $p$  always

N  $p \ \& \ \sim p$

This is necessarily false

Y  $p \ \& \ q$

Y  $\sim(\sim p \vee \sim q)$

Both are conjunctions. Good bet!

Y  $p \rightarrow q$

N  $q \vee \sim q$

It's not a conditional, but if you double up on an occurrence of one part of the sentence, you can make this work. See how?

This is necessarily true.

Y  $q$  always

Y  $p \vee q$

Same answer as for  $p \rightarrow q$

Y  $(p \rightarrow q) \ \& \ \sim q$

Y  $(p \rightarrow q) \ \& \ p$

The three messy conjunctions (above and below right) have conditionals as parts. Good bet. Just sort out the negations.

N  $p \rightarrow (q \vee r)$

Y  $p \ \& \ (\sim q \vee r)$

not a conditional

*In class, we saw that there are many ways to schematize any sentence accurately, even if not all ways of doing so are particularly useful. Conversely, any schema accurately schematizes indefinitely many sentences. For each of the following sentences, indicate whether (Y/N) it is accurately schematized by the propositional form:*

*First, try to manipulate the form so you can more readily recognize it when it shows up in the sentences.*

$$\begin{aligned} \neg(p \vee (\neg q \ \& \ \neg r)) &= \neg p \ \& \ (q \vee r) = \neg p \ \& \ (\neg q \ \rightarrow \ r) = \\ &(\neg p \ \& \ q) \vee (\neg p \ \& \ r) \end{aligned}$$

Y *If the storm strikes, we will be flooded or blown away.*

*To make this one work, you would have to make p correspond to something like “the storm strikes and it does not strike”, which, when negated, is true. Then you can make q correspond to “it is not the case that the storm strikes” and r correspond to “we will be flooded or blown away”. But these gymnastics involving p make this schematization particularly unhelpful.*

Y *Faculty may not attend, but students or staff may.*

*There are two ways to read this sentence. The natural way reads ‘or’ as having the force of ‘and’ above. So, we could make p correspond to “faculty may attend”, q correspond to “students or staff may attend”, and r correspond to something like “students may attend and may not attend” which must be false. This is, like the first one, an unhelpful schematization.*

*The less natural way to read the sentence reads ‘or’ as having the logical force of ‘or’, in which case the second permutation of the form above works quite straightforwardly.*

Y *It is cloudy, but if the wind doesn’t pick up, we will sail.*

*This one is fairly easily seen to be an example of the form.*

Y *It’s not true that either we go now or are late and tired.*

*This one was most obviously like the original form.*

\_\_Y\_ Either we will win, we will lose, or we will refuse to play.

*This can be made to work, but with gymnastics akin to those needed for the first answer. Can you see why?*

\_\_Y\_ He is tanned, rested, and ready.

*Yes, but with gymnastics like that involved in the second answer. See why?*

\_\_Y\_ He is ready only if he is tanned and rested.

\_\_Y\_ He won the game despite being distracted and hungry.

*Both of these last answers require gymnastics that make the form an unhelpful way to schematize them.*

*The lesson? It's relatively easy to shoehorn a sentence into a propositional form, if you're willing to include a number of parts that are either necessarily true or necessarily false, and if you're willing to allow a part to appear more than once when it only appears once in the form (or sentence). This doesn't mean that any form properly schematizes any sentence, or conversely, for any assignment of parts to letters, of course. Schematizing is useful, but only when it is done in a way that helps you understand the structures of the statements you schematize.*

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