Homework #6

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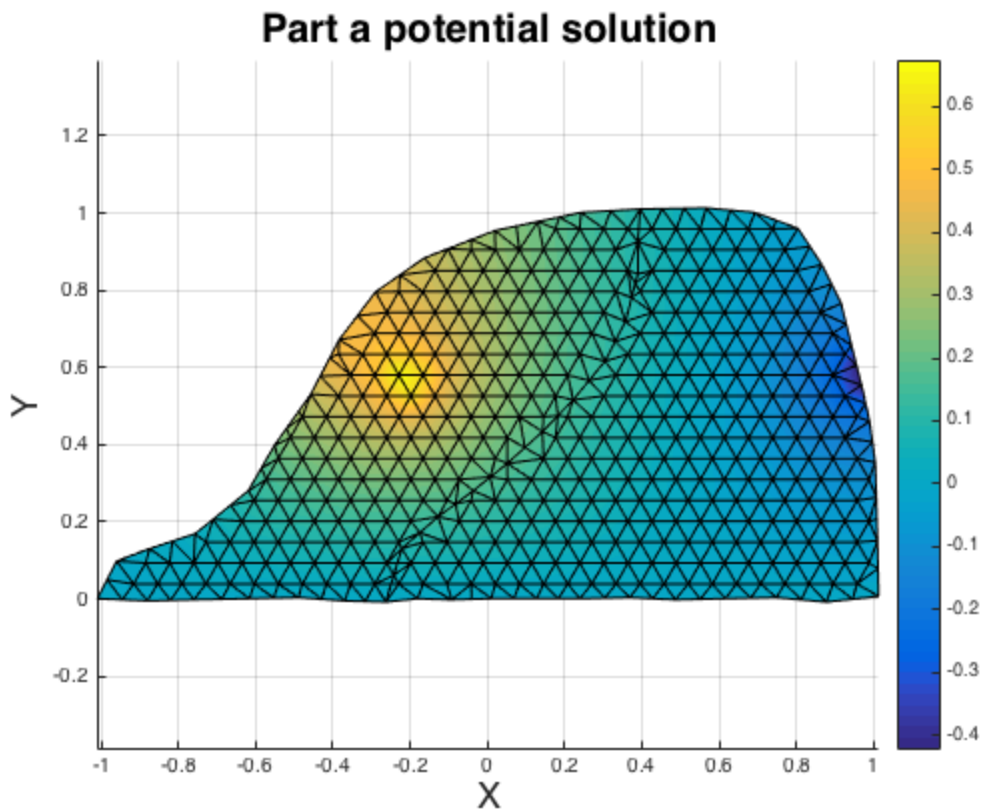
Part A

```
%Prepare the workspace
% Read in data
clearvars
ele=dlmread('hw44.ele.txt');
nod=dlmread('hw44.nod.txt');
bc=dlmread('hw44.dnd.txt');
pd=dlmread('hw44.bel.txt');
% define parameters
x=nod(:,2);
y=nod(:,3);
x_0=nod(503,2);
y_0=nod(503,3);
thick=1+x./10+y./5;
% assemble matrix
a_mat=zeros(length(nod)-3);
b_ary=zeros(length(nod)-3,1);
for k=1:length(ele)
    e_nod=ele(k,2:4);
    x_e=nod(e_nod,2);
    y_e=nod(e_nod,3);
    t_e=thick(e_nod);
    delt_x(1)=x_e(2)-x_e(3);
    delt_x(2)=x_e(3)-x_e(1);
    delt_x(3)=x_e(1)-x_e(2);
    delt_y(1)=y_e(2)-y_e(3);
    delt_y(2)=y_e(3)-y_e(1);
    delt_y(3)=y_e(1)-y_e(2);
    a_area=0.5*(x_e(1)*delt_y(1)+x_e(2)*delt_y(2)+x_e(3)*delt_y(3));
    for i=1:3
        for j=1:3
            amat_e(i,j)=-(delt_x(i)*delt_x(j)+delt_y(i)*delt_y(j))/
(4*a_area)*t_e(i);

 a_mat(e_nod(i),e_nod(j))=a_mat(e_nod(i),e_nod(j))+amat_e(i,j);
        end
```

```
    end
end
% modify A matrix and B array base on the boundary condition
a_mat(bc(:,1),:)=0;
for i=1:length(bc)
    a_mat(bc(i,1),bc(i,1))=1;
end
b_ary(bc(:,1))=0;
b_ary(492)=0.25;
b_ary(493)=0.25;
% modify B array according to the source
x_s=x(ele(288,2:4));
y_s=y(ele(288,2:4));
delt_x(1)=x_s(2)-x_s(3);
delt_x(2)=x_s(3)-x_s(1);
delt_x(3)=x_s(1)-x_s(2);
delt_y(1)=y_s(2)-y_s(3);
delt_y(2)=y_s(3)-y_s(1);
delt_y(3)=y_s(1)-y_s(2);
a_area=0.5*(x_s(1)*delt_y(1)+x_s(2)*delt_y(2)+x_s(3)*delt_y(3));
b_ary(ele(288,2))=-((x_s(2)*y_s(3)-x_s(3)*y_s(2))+delt_y(1)*x_0-
delt_x(1)*y_0)/(2*a_area);
b_ary(ele(288,3))=-((x_s(3)*y_s(1)-x_s(1)*y_s(3))+delt_y(2)*x_0-
delt_x(2)*y_0)/(2*a_area);
b_ary(ele(288,4))=-((x_s(1)*y_s(2)-x_s(2)*y_s(1))+delt_y(3)*x_0-
delt_x(3)*y_0)/(2*a_area);
u=a_mat\b_ary;
% plot result
figure()
for i=1:size(ele,1)

 patch(nod(ele(i,2:4),2),nod(ele(i,2:4),3),u(ele(i,2:4)),'FaceColor','interp');
    hold on;
end
c= colorbar;
colormap;
title('Part a potential solution','fontsize',20)
xlabel('X','fontsize',18)
ylabel('Y','fontsize',18)
axis equal;
grid on;
```

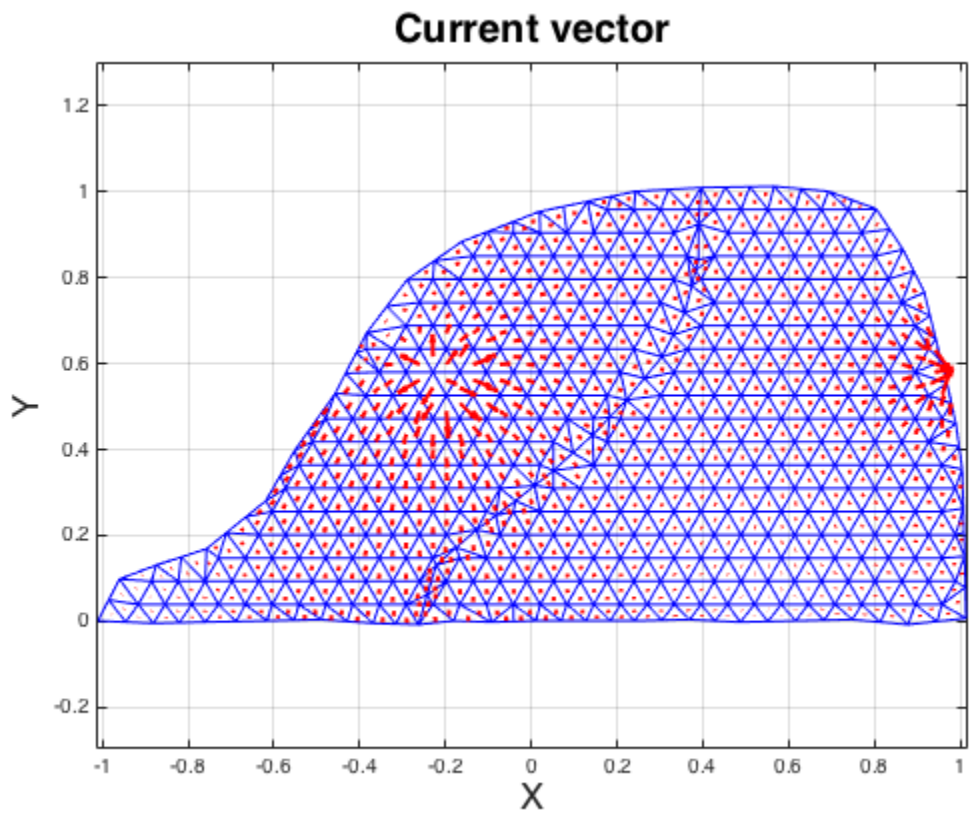


Part B

calculate the current vector

```
for k=1:length(ele)
    e_nod=ele(k,2:4);
    x_e=nod(e_nod,2);
    y_e=nod(e_nod,3);
    x_lab(k)=mean(x_e);
    y_lab(k)=mean(y_e);
    delt_x(1)=x_e(2)-x_e(3);
    delt_x(2)=x_e(3)-x_e(1);
    delt_x(3)=x_e(1)-x_e(2);
    delt_y(1)=y_e(2)-y_e(3);
    delt_y(2)=y_e(3)-y_e(1);
    delt_y(3)=y_e(1)-y_e(2);
    a_area=0.5*(x_e(1)*delt_y(1)+x_e(2)*delt_y(2)+x_e(3)*delt_y(3));
    fl=u(e_nod);
    v_x(k)=fl(1)*delt_y(1)/(2*a_area)+fl(2)*delt_y(2)/
(2*a_area)+fl(3)*delt_y(3)/(2*a_area);
    v_y(k)=(fl(1)*delt_x(1)/(2*a_area)+fl(2)*delt_x(2)/
(2*a_area)+fl(3)*delt_x(3)/(2*a_area));
end
figure()
triplot([ele(:,2),ele(:,3),ele(:,4)],nod(:,2),nod(:,3));
hold on
```

```
%trisurf([ele(:,2),ele(:,3),ele(:,4)],nod(:,2),nod(:,3),u,'FaceColor','interp');
quiver(x_lab,y_lab,-v_x,v_y,'linewidth',2,'color','red')
title('Current vector','fontsize',20)
xlabel('X','fontsize',18)
ylabel('Y','fontsize',18)
axis equal;
grid on;
```

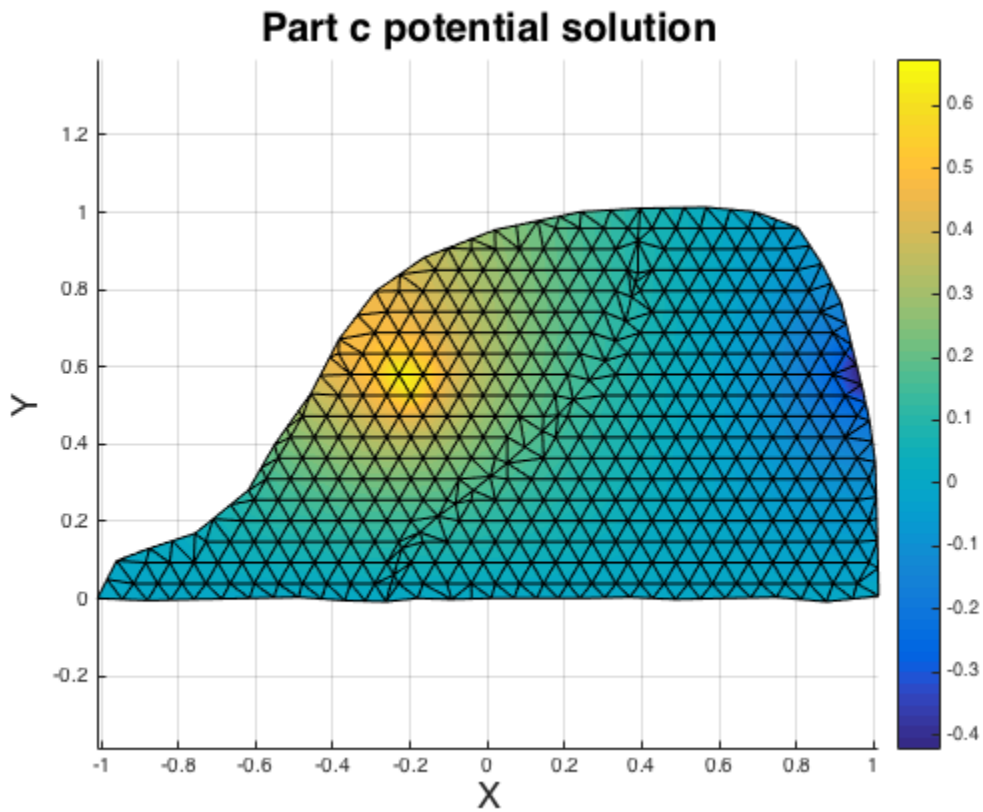


Part C

using LU to solve the problem

```
[low,upp]=lu(a_mat);
y_ary=low\b_ary;
u_lu=upp\y_ary;
% plot result
figure()
for i=1:size(ele,1)

 patch(nod(ele(i,2:4),2),nod(ele(i,2:4),3),u_lu(ele(i,2:4)),'FaceColor','interp');
    hold on;
end
c= colorbar;
colormap;
title('Part c potential solution','fontsize',20)
xlabel('X','fontsize',18)
```

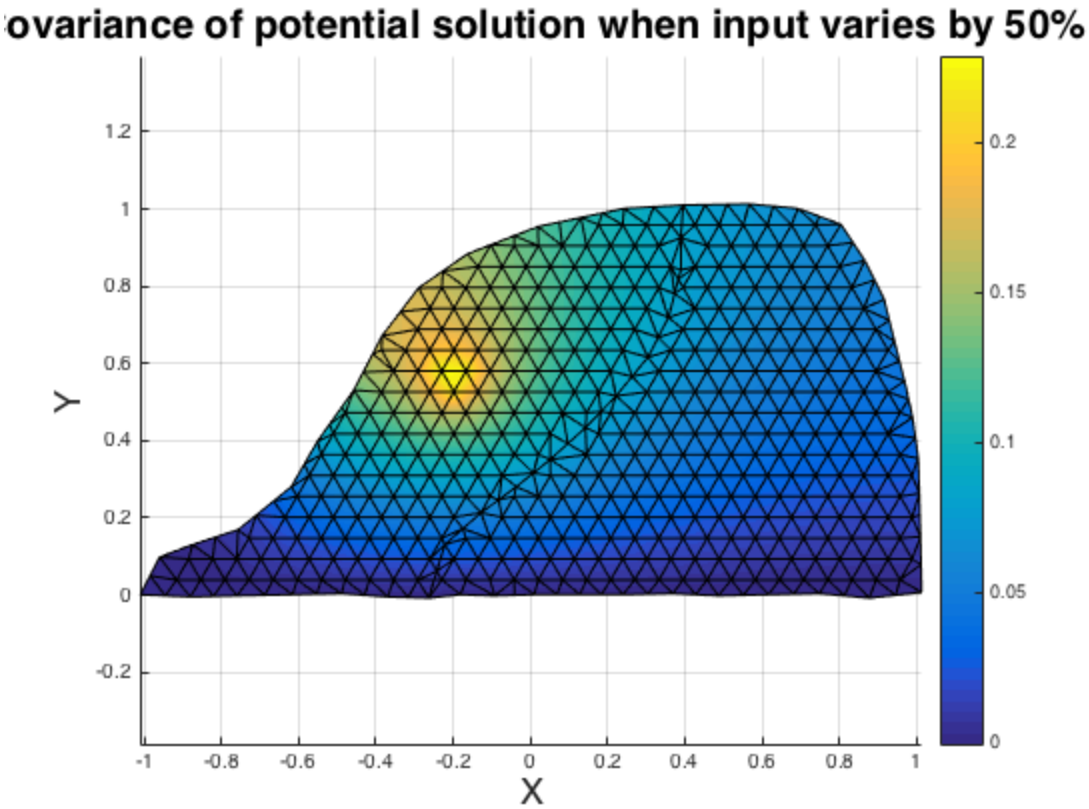
```
ylabel('Y','fontsize',18)
axis equal;
grid on;
```



Part D: 1

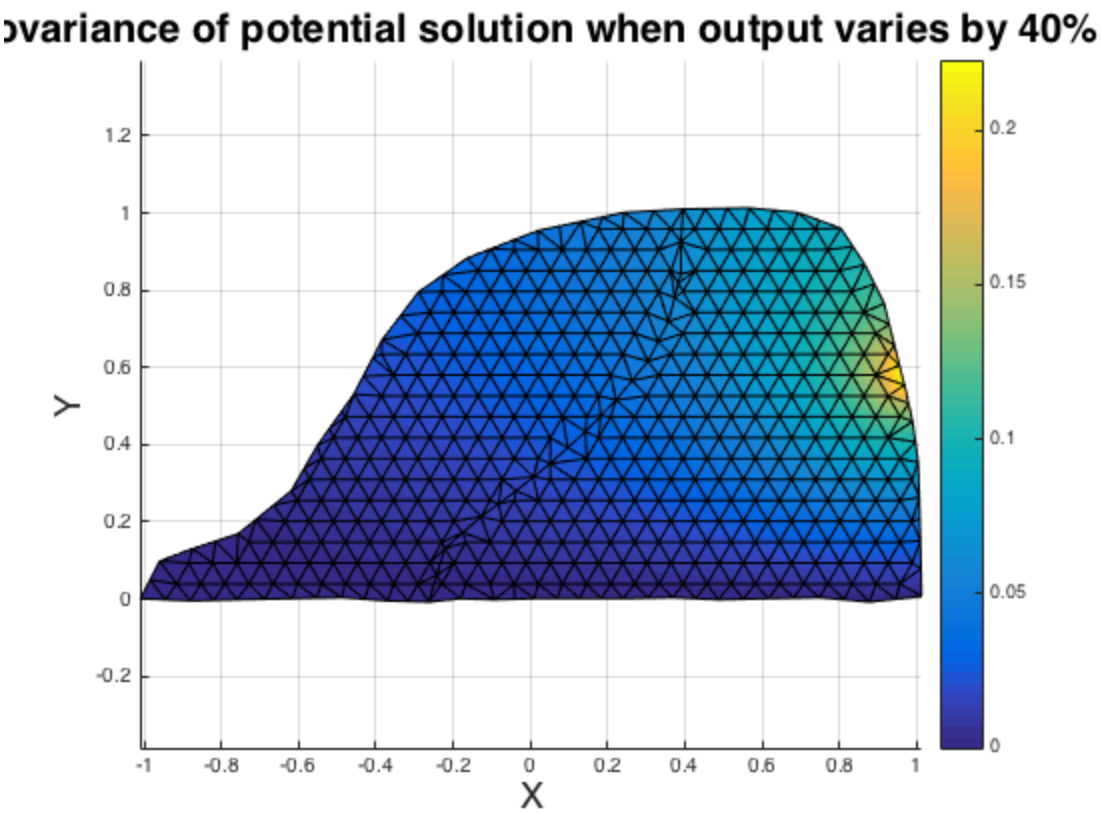
```
i_b_ary=zeros(502,1);
i_b_ary(ele(288,2))=b_ary(ele(288,2))*0.5;
i_b_ary(ele(288,3))=b_ary(ele(288,2))*0.5;
i_b_ary(ele(288,4))=b_ary(ele(288,2))*0.5;
i_conv_u=inv(a_mat)*i_b_ary*(i_b_ary.')*(inv(a_mat).');
i_u=sqrt(diag(i_conv_u));
% plot result
figure()
for i=1:size(ele,1)

 patch(nod(ele(i,2:4),2),nod(ele(i,2:4),3),i_u(ele(i,2:4)),'FaceColor','interp');
    hold on;
end
c= colorbar;
colormap;
title('Covariance of potential solution when input varies by
 50%','fontsize',20)
xlabel('X','fontsize',18)
ylabel('Y','fontsize',18)
axis equal;
```

```
grid on;
```



Part D: 2

```
ii_b_ary=zeros(502,1);
ii_b_ary(492)=0.4*b_ary(492);
ii_b_ary(493)=0.4*b_ary(493);
ii_conv_u=inv(a_mat)*ii_b_ary*(ii_b_ary.')*(inv(a_mat).');
ii_u=sqrt(diag(ii_conv_u));
% plot result
figure()
for i=1:size(ele,1)

 patch(nod(ele(i,2:4),2),nod(ele(i,2:4),3),ii_u(ele(i,2:4)),'FaceColor','interp');
    hold on;
end
c= colorbar;
colormap;
title('Covariance of potential solution when output varies by
 40%','fontsize',20)
xlabel('X','fontsize',18)
ylabel('Y','fontsize',18)
axis equal;
grid on;
```

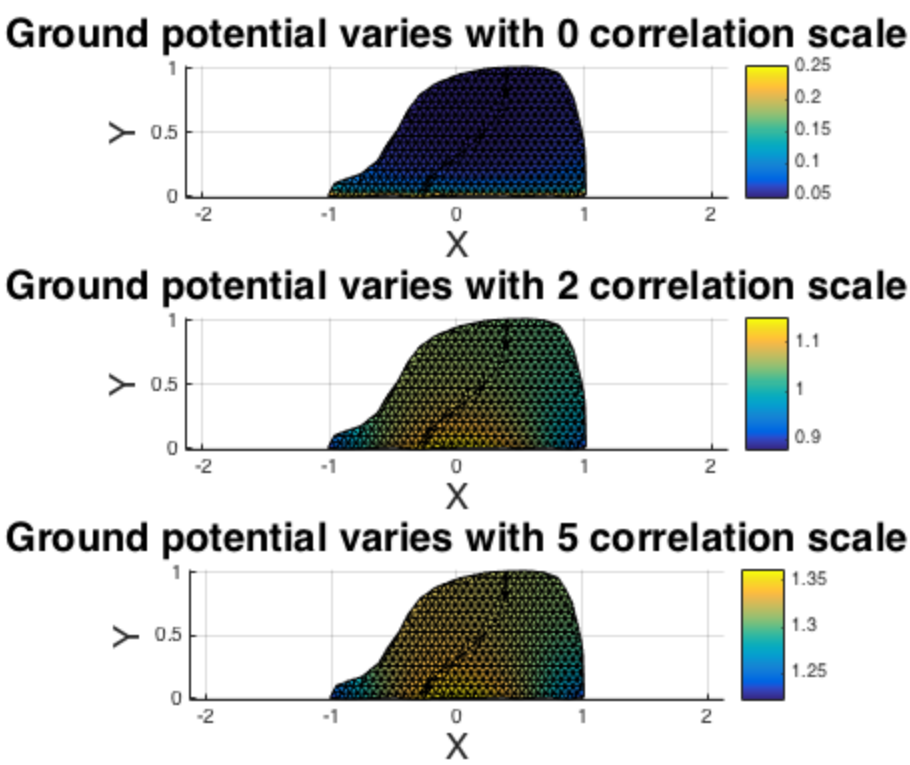


Part D: 3

```
iii_b_0=zeros(502);
sig=0.5;
l=1e-10;
for i=1:length(bc)
    bc_i=bc(i,1);
    x_i=nod(bc_i,2);
    y_i=nod(bc_i,3);
    for j=1:length(bc)
        bc_j=bc(j,1);
        x_j=nod(bc_j,2);
        y_j=nod(bc_j,3);
        r_ij=sqrt((x_i-x_j)^2+(y_i-y_j)^2)*6.5/2;
        bibj=sig^2*(1+r_ij/l)*exp(-r_ij/l);
        iii_b_0(bc_i,bc_j)=bibj;
    end
end
iii_conv_u_0=inv(a_mat)*iii_b_0*iii_b_0.'*(inv(a_mat).');
iii_u_0=sqrt(diag(iii_conv_u_0));
% plot result
figure()
subplot(3,1,1)
for i=1:size(ele,1)
```

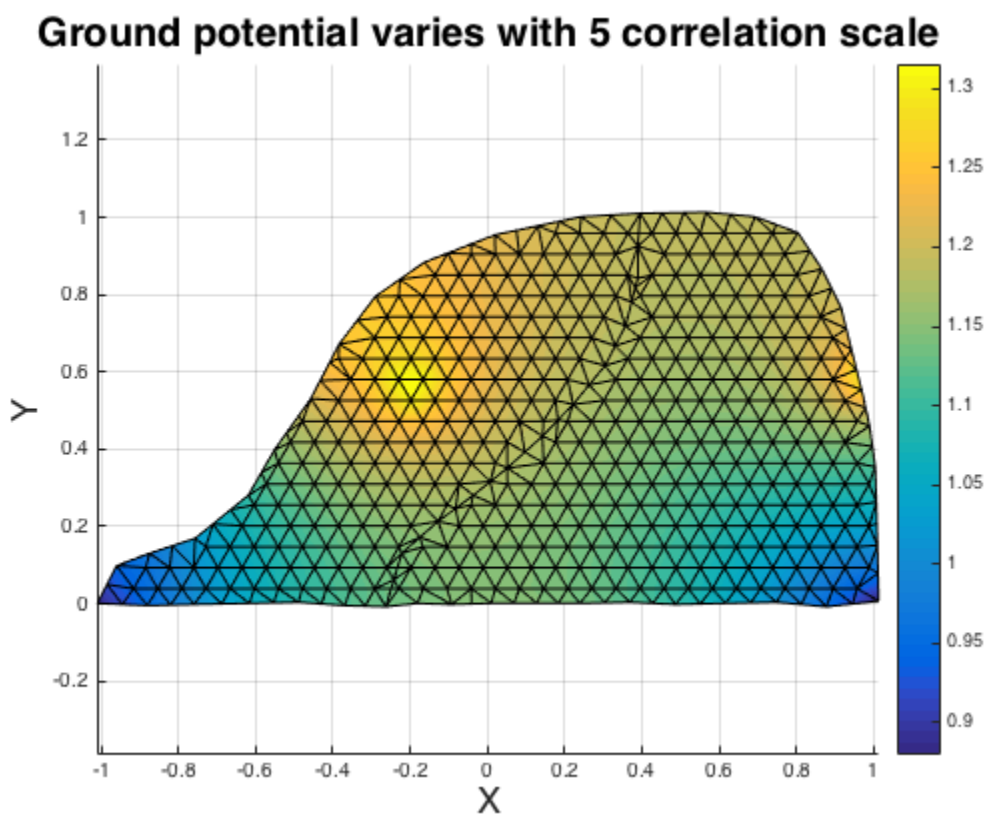
```
 patch(nod(ele(i,2:4),2),nod(ele(i,2:4),3),iii_u_0(ele(i,2:4)),'FaceColor','interp
    hold on;
end
c= colorbar;
colormap;
title('Ground potential varies with 0 correlation
 scale','fontsize',20)
xlabel('X','fontsize',18)
ylabel('Y','fontsize',18)
axis equal;
grid on;
%
iii_b_2=zeros(502);
sig=0.5;
l=2;
for i=1:length(bc)
    bc_i=bc(i,1);
    x_i=nod(bc_i,2);
    y_i=nod(bc_i,3);
    for j=1:length(bc)
        bc_j=bc(j,1);
        x_j=nod(bc_j,2);
        y_j=nod(bc_j,3);
        r_ij=sqrt((x_i-x_j)^2+(y_i-y_j)^2)*6.5/2;
        bibj=sig^2*(1+r_ij/l)*exp(-r_ij/l);
        iii_b_2(bc_i,bc_j)=bibj;
    end
end
iii_conv_u_2=inv(a_mat)*iii_b_2*iii_b_2.'*(inv(a_mat).');
iii_u_2=sqrt(diag(iii_conv_u_2));
% plot result
subplot(3,1,2)
for i=1:size(ele,1)

 patch(nod(ele(i,2:4),2),nod(ele(i,2:4),3),iii_u_2(ele(i,2:4)),'FaceColor','interp
    hold on;
end
c= colorbar;
colormap;
title('Ground potential varies with 2 correlation
 scale','fontsize',20)
xlabel('X','fontsize',18)
ylabel('Y','fontsize',18)
axis equal;
grid on;
%
iii_b_5=zeros(502);
sig=0.5;
l=5;
for i=1:length(bc)
    bc_i=bc(i,1);
    x_i=nod(bc_i,2);
    y_i=nod(bc_i,3);
```

```
    for j=1:length(bc)
        bc_j=bc(j,1);
        x_j=nod(bc_j,2);
        y_j=nod(bc_j,3);
        r_ij=sqrt((x_i-x_j)^2+(y_i-y_j)^2)*6.5/2;
        bibj=sig^2*(1+r_ij/l)*exp(-r_ij/l);
        iii_b_5(bc_i,bc_j)=bibj;
    end
end
iii_conv_u_5=inv(a_mat)*iii_b_5*iii_b_5.'*(inv(a_mat).');
iii_u_5=sqrt(diag(iii_conv_u_5));
% plot result
subplot(3,1,3)
for i=1:size(ele,1)

 patch(nod(ele(i,2:4),2),nod(ele(i,2:4),3),iii_u_5(ele(i,2:4)),'FaceColor','interp
    hold on;
end
c= colorbar;
colormap;
title('Ground potential varies with 5 correlation
 scale','fontsize',20)
xlabel('X','fontsize',18)
ylabel('Y','fontsize',18)
axis equal;
grid on;
```



part D: 4

```
iv_u=iii_u_2+ii_u+i_u;
% plot result
figure()
for i=1:size(ele,1)

 patch(nod(ele(i,2:4),2),nod(ele(i,2:4),3),iv_u(ele(i,2:4)),'FaceColor','interp');
    hold on;
end
c= colorbar;
colormap;
title('Ground potential varies with 5 correlation
 scale','fontsize',20)
xlabel('X','fontsize',18)
ylabel('Y','fontsize',18)
axis equal;
grid on;
```



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