Contents


```
a=0.1;
b=1;
i0=1;
kap=3;
n=80;
h=(b-a)/(n-1);
k=pi/2/(n-1);
beta=h^2/k^2;
a_mat=zeros((n-1)^2);
b_ary=zeros((n-1)^2,1);
b_ary_1=zeros((n-1)^2,1);
b_ary_2=zeros((n-1)^2,1);
% for zz=1:1000
count=0;
a_var=100;
for i=1:n
    for j=1:n-1
        count=count+1;
        theta=(i-1)*k;
        r=(j-1)*h+a;
        r_ray(count)=r;
        theta=(i-1)*((pi/2)/(n-1));
        theta_ray(count)=theta;
        a_mat((i-1)*(n-1)+j,(i-1)*(n-1)+j)=-2*(1+beta/r^2);
        if i==1 %boundry C
            if j==1 %corner BC
                a_mat((i-1)*(n-1)+j,(i-1)*(n-1)+j+1)=2;
                a_mat((i-1)*(n-1)+j,i*(n-1)+j)=2*beta/r^2;
            elseif j==(n-1) % corner CD
                a_mat((i-1)*(n-1)+j,i*(n-1)+j)=2*beta/r^2;
                a_mat((i-1)*(n-1)+j,(i-1)*(n-1)+j-1)=(1-h/(2*r));
            else
                a_mat((i-1)*(n-1)+j,i*(n-1)+j)=2*beta/r^2;
                a_mat((i-1)*(n-1)+j,(i-1)*(n-1)+j-1)=(1-h/(2*r));
                a_mat((i-1)*(n-1)+j,(i-1)*(n-1)+j+1)=(1+h/(2*r));
            end
        end
        if j==(n-1) %boundry D
            if i==1 %corner CD
                a_mat((i-1)*(n-1)+j,i*(n-1)+j)=2*beta/r^2;
```

```
            a_mat((i-1)*(n-1)+j,(i-1)*(n-1)+j-1)=(1-h/(2*r));
        elseif i== n %corner AD
            a_mat((i-1)*(n-1)+j,(i-1)*(n-1)+j)=-2*(1+beta/r^2)+2*k*a_var*beta/r^2;
            a_mat((i-1)*(n-1)+j,(i-2)*(n-1)+j)=2*beta/r^2;
            a_mat((i-1)*(n-1)+j,(i-1)*(n-1)+j-1)=(1-h/(2*r));
        else
            a_mat((i-1)*(n-1)+j,i*(n-1)+j)=beta/r^2;
            a_mat((i-1)*(n-1)+j,(i-2)*(n-1)+j)=beta/r^2;
            a_mat((i-1)*(n-1)+j,(i-1)*(n-1)+j-1)=(1-h/(2*r));
        end
    end
    if j==1 %boundry B
        if i==1 %corner BC
            a_mat((i-1)*(n-1)+j,(i-1)*(n-1)+j+1)=2;
            a_mat((i-1)*(n-1)+j,i*(n-1)+j)=2*beta/r^2;
        elseif i==n %corner AB
            a_mat((i-1)*(n-1)+j,(i-1)*(n-1)+j)=-2*(1+beta/r^2)+2*k*a_var*beta/r^2;
            a_mat((i-1)*(n-1)+j,(i-1)*(n-1)+j+1)=2;
            a_mat((i-1)*(n-1)+j,(i-2)*(n-1)+j)=2*beta/r^2;
        else
            a_mat((i-1)*(n-1)+j,(i-1)*(n-1)+j+1)=2;
            a_mat((i-1)*(n-1)+j,(i-2)*(n-1)+j)=beta/r^2;
            a_mat((i-1)*(n-1)+j,(i)*(n-1)+j)=beta/r^2;
        end
    end
    if i==n %boundry A
        if j==1 %corner AB
            a_mat((i-1)*(n-1)+j,(i-1)*(n-1)+j)=-2*(1+beta/r^2)+2*k*a_var*beta/r^2;
            a_mat((i-1)*(n-1)+j,(i-1)*(n-1)+j+1)=2;
            a_mat((i-1)*(n-1)+j,(i-2)*(n-1)+j)=2*beta/r^2;
        elseif j==(n-1) %corner AD
            a_mat((i-1)*(n-1)+j,(i-1)*(n-1)+j)=-2*(1+beta/r^2)+2*k*a_var*beta/r^2;
            a_mat((i-1)*(n-1)+j,(i-2)*(n-1)+j)=2*beta/r^2;
            a_mat((i-1)*(n-1)+j,(i-1)*(n-1)+j-1)=(1-h/(2*r));
        else
            a_mat((i-1)*(n-1)+j,(i-1)*(n-1)+j)=-2*(1+beta/r^2)+2*k*a_var*beta/r^2;
            a_mat((i-1)*(n-1)+j,(i-1)*(n-1)+j+1)=(1+h/(2*r));
            a_mat((i-1)*(n-1)+j,(i-1)*(n-1)+j-1)=(1-h/(2*r));
            a_mat((i-1)*(n-1)+j,(i-2)*(n-1)+j)=beta/r^2;
        end
    end
    if i~=1 && i~=n && j~=1 && j~=(n-1)
        a_mat((i-1)*(n-1)+j,(i-1)*(n-1)+j+1)=(1+h/(2*r));
        a_mat((i-1)*(n-1)+j,(i-1)*(n-1)+j-1)=(1-h/(2*r));
        a_mat((i-1)*(n-1)+j,(i-2)*(n-1)+j)=beta/r^2;
        a_mat((i-1)*(n-1)+j,(i)*(n-1)+j)=beta/r^2;
    end
    if j==(n-1)
        b_ary_1((i-1)*(n-1)+j,1)=(1+h/(b-h))*(-i0*b*cos(3.*theta));
    end
    b_ary=-b_ary_1;
  end
end
u=a_mat\b_ary;
%uu(zz,:)=u;
u_mat=reshape(u,[(n-1),(n)]);
[x_ray,y_ray]=pol2cart(theta_ray,r_ray);
```

```
x_mat=reshape(x_ray,[n-1,(n)]);
y_mat=reshape(y_ray,[n-1,(n)]);
%end
% analytic solution
analy= @(th,rr) -(rr.^3+0.1.^6.*rr.^-3)./(1+0.1.^6).*cos(3.*th);
ana_u=analy(theta_ray,r_ray);
ana_mat=reshape(ana_u,[n-1,(n)]);
```

```
for yy=1:1000
    err_u_1(yy)=sum(abs(uu(yy,:)-uu(1,:)));
end
figure()
plot(0:0.001:0.9,err_u_1(1:901),'linewidth',2)
title('Error to Type II BC','fontsize',25)
xlabel('A value','fontsize',20)
ylabel('Error','fontsize',20)
```

```
Undefined variable uu.

Error in hw2p1 (line 108)
    err_u_1(yy)=sum(abs(uu(yy,:)-uu(1,:)));
```

for n=5

```
err_5=reshape(u'-ana_u,[(n-1),(n-1)]);
```

for n=10

```
err_10=reshape(u'-ana_u,[(n-1),(n-1)]);
```

for n=50

```
err_50=reshape(u'-ana_u,[(n-1),(n-1)]);
```

for n=100

```
err_100=reshape(u'-ana_u,[(n-1),(n-1)]);
```

for n=150

```
err_150=reshape(u'-ana_u,[(n-1),(n-1)]);
```

for n=200

```
err_200=reshape(u'-ana_u,[(n-1),(n-1)]);
```

compare RMS

```
rms_err(1)=rms(err_5(:));
rms_err(2)=rms(err_10(:));
rms_err(3)=rms(err_50(:));
rms_err(4)=rms(err_100(:));
rms_err(5)=rms(err_200(:));
```

visualization for the solution

```
[xx,yy]=meshgrid(0:0.0001:1.2);
yy=(yy-1.2).*(-1);
mask=zeros(size(xx));
mask=(xx.^2+yy.^2).^0.5;
mask_t=ones(size(mask));
mask_t(mask<0.1)=NaN;
mask_t(mask>1)=NaN;
for i=1:size(x_ray,2)
    val_mask(:,:,i)=abs(xx-x_ray(i))+abs(yy-y_ray(i));
end
[x_min,x_idx]=min(val_mask,[],3);
for i=1:size(x_idx,1)
    for j=1:size(x_idx,2)
        map(i,j)=u(x_idx(i,j));
    end
end
map_c=map.*mask_t;
figure()
imagesc(map_c)
```

contour plot

```
figure()
pcolor(x_mat,y_mat,u_mat)
% hold on
%contour(x_mat,y_mat,u_mat)
title('Matrix Solution A=1','fontsize',25)
figure()
pcolor(x_mat,y_mat,ana_mat)
title('Analytic Solution A=1','fontsize',25)
```

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