3: reason and argument

Probability review problems with answers

Let's say we have a test for some disease that is 95% selective. You do NOT have the disease. How many times would you have to take the test in order to make it more likely than not that you will return at least one positive result despite not having the disease?

We need to know: Pr(p|-d), or the probability that you test positive given you don't have the disease. According to the selectivity, Pr(p|-d) = 1/20. The probability that you test positive in at least one of n tries is 1 minus the probability that you fail to test positive in n tries:

Pr(one positive result in n tries) = 1 – $[\Pr(-p|-d)]^n$ = 1 – $[19/20]^n$

For n = 14: Pr(one positive result in n tries) = 1 – .49 = .51

Let's say 10% of the population can safely donate blood to you. We have a test for blood type that is 90% selective and 90% sensitive. Someone tests positive for being a donor. What is the probability that she is a donor?

We need to know: $Pr(d|p)$, or the probability that someone is a donor given that she has tested positive for being one. We know that $Pr(p|-d)$ is 1/10, because the test is 90% selective. Put differently, the probability that you test positive for being a donor, even though you are not, is 1 in 10. We also know that $Pr(p|d)$ is 9/10, since the test is 90% sensitive: 90% of donors are correctly classified as such by the test.

Let's apply Bayes' Rule:

$Pr(d|p) = Pr(d)Pr(p|d)/Pr(p) =$
$[(1/10)(9/10)]/Pr(p)$ [see below] $=$
$(9/100)/(18/100) = 1/2$

$Pr(p) = Pr(p\&d \vee p\&-d) = Pr(p\&d) + Pr(p\&-d) - 0 = Pr(d)Pr(p|d) + Pr(-d)Pr(p|-d) =$
$(1/10)(9/10) + (9/10)(1/10) = 18/100$

Two people named ‘Lauren’, Lauren A and Lauren B, each order three burritos at Boloco. Lauren A orders three Tofu Teriyaki while Lauren B orders two Tofu Teriyaki and one Beef Bangkok. “Lauren!” they yell, showing two bags, with three burritos each. Lauren A takes a TT burrito out of a bag and proclaims: “This one is probably mine.” Lauren B objects: “You can’t know that. I ordered TT burritos too.”

(a) Is Lauren A right, i.e., is Pr(The bag is Lauren A’s bag|she found a TT burrito in it) > ½? What is the specific value?
Yes, 3/5

(b) Lauren A then pulls another TT burrito from the bag. What is Pr(Bag A is Lauren A’s bag |She pulls two TT burritos from it)?
3/4

There are five TT burritos in all between the two bags. In three of those cases, the bag is Lauren A’s while in two it is Lauren B’s, so Lauren A is right and the probability that the bag is hers is 3/5.

How many ways are there to draw two TT burritos from a bag? Let’s think about it in two ways.

First, we imagine Lauren A drawing two burritos at once. There are three ways of doing so: TT1&TT2, TT2&TT3, TT1&TT3. For Lauren B’s bag there is one way of doing so, because there are only two TT burritos in her bag. In 3 out of 4 cases, the bag is Lauren A’s.

Second, imagine Lauren A first drawing one burrito, and then drawing another. There are eight possible sequences:
If she has her own bag, she could draw:
1 then 2, 1 then 3, 2 then 1, 2 then 3, 3 then 1, 3 then 2
If she has Lauren B’s bag, she could draw:
1 then 2, 2 then 1
Here out of 8 possibilities, 6 of them are Lauren A’s bag, so the probability if the same: 3/4.

Monty Hall has a new game. It's just like the old Make-a-deal scenario: 3 doors, two goats, one Maserati. You choose a door, say door A. Here's the new part: he then flips a coin. If it comes up heads, he opens door B, if it comes up tails he opens door C. Let's say it comes up heads and he opens door B, revealing a goat. He then gives you the chance to switch your choice from A to C. Is it to your advantage to switch? Explain.

No. Your odds are even. The coin flip prevents him from avoiding the car. So, if need be he must reveal the car before allowing you the choice to switch. Given that, you cannot take the door he opens to be evidence of whether there is or is not a car behind the other door. We worked this out with the equations in class.

It rains a lot in the Pacific Northwest, especially in the winter: 2 out of 3 days in winter are rainy, while just one is sunny. In the summer, 2 out of 3 days are sunny and just one is rainy. You wake up from a coma in Twin Peaks, Washington. You know that it's either winter or summer, and you can assume Pr(winter) = ½ = Pr(summer). You noticed that the past three days have been sunny, what is the probability that it is winter? Show your work and clearly indicate your answer.

We want to know: Pr(winter|the last three days were sunny)
Abbreviating: **Pr(w|3)** = Pr(3|w)Pr(w)/Pr(3)

Pr(3) = Pr[(3 & w) v (3 & -w)] = Pr(3 & w) + Pr(3 & -w)
= Pr(w)Pr(3|w) + Pr(-w)Pr(3|-w) = ½ (Pr(3|w) + Pr(3|-w))

Pr(3|w) = [Pr(a day is sunny in winter)]^3 = (1/3)^3 = 1/27
Pr(3|-w) = [Pr(a day is sunny in summer)]^3 = (2/3)^3 = 8/27

So, Pr(3) = ½ (1/27 + 8/27) = ½ (1/3) = 1/6.
And,
Pr(w|3) = Pr(3|w)Pr(w)/Pr(3) = (1/27)(1/2)/(1/6) = **1/9**

Interesting mistake:
Calculate Pr(it's sunny on a day) = Pr(s) =Pr[(s&w) v (s&-w)] = (1/3*1/2)+(2/3*1/2) = ½. Then say: Pr(3) = Pr(s)^3 = 1/8.
Why is that a mistake? This gives the probability that three days, taken at random throughout the summer and winter, are sunny, not the probability that three *consecutive* days are sunny, all in the winter, or in the summer, which is what you need. Always attend to the statements whose probabilities you seek!

Let's say 5% of nickels are unfair, in that when flipped they come up heads 60% of the time. The other 95% of nickels are fair. You flip a nickel 10 times, and every flip is heads. What is the probability that your nickel is unfair?

Pr(h|f) = ½
Pr(h|-f) = 3/5
Pr(f) = 19/20
Pr(-f) = 1/20

Pr(no tails in 10 tries|f) = $[\Pr(h|f)]^{10} = (1/2)^{10}$ = 1/1024
Pr(no tails in 10 tries|-f) = $[\Pr(h|-f)]^{10} = (3/5)^{10}$ = Pr(no tails in 10 tries)Pr(-f|no tails in 10 tries)/Pr(-f)
That last part applies Bayes' rule.

And now we solve for the value of interest:
Pr(-f|no tails in 10 tries) = $(3/5)^{10}(1/20)$/Pr(no tails in 10 tries) = .0003/Pr(no tails in 10 tries)

The last piece of the puzzle:
Pr(no tails in 10 tries) =
Pr((no tails in 10 tries & f) v (no tails in 10 tries & -f)) =
Pr(no tails in 10 tries & f) + Pr(no tails in 10 tries & -f) – 0 =
Pr(f)Pr(no tails in 10 tries|f) +Pr(-f)Pr(no tails in 10 tries|-f)
=(19/20)(1/1024) + $(1/20)(3/5)^{10}$ = .0009 + .0003 = .0012

Pr(-f|no tails in 10 tries) = .0003/.0012 = .25 = 25%

NB: I will not ask you to solve problems that require extensive calculation, as this one does, on the quiz. For the quiz, fractions will suffice.

Half the time, your gas gauge is accurate, but half the time, it just says you have ½ a tank of gas, regardless of whether that's true. The Dude has just returned your car. 70% of the time, he returns it with an almost empty tank, while 30% of the time, he returns it half full. You get in your car and see the gauge indicating ½ tank. In these circumstances, what is the probability that the tank is half full? Show your work and clearly indicate your answer.

The Dude returns the car 100 times. 30 of those times the tank is half full, and the gauge reads ½, because it will do so even if it is broken (people forget this point). 70 of those times the tank is almost empty. In 35 of those cases, the gauge reads half full anyway because it's broken. So, in 100 returns, the gauge reads half full 65 times, and in 30 of those cases it really is half full. The probability that it is half full, given the reading and the fact that The Dude just returned it, then, is 30/65, or 6/13.

We want to know:
Pr(tank is half full|gauge reads half) = **Pr(h|r)**=Pr(r|h)Pr(h)/Pr(r)

We know: Pr(r|h) = 1; Pr(h) = 3/10 (That's The Dude, for ya.)

Pr(r) = Pr[(r & h) v (r v -h)] = Pr(r|h)Pr(h) + Pr(r|-h)Pr(-h) =
3/10 + ½ (7/10) = 13/20
Many thought that Pr(r/h) = ½, but a broken gauge reads ½ tank!

Pr(h|r) = Pr(r|h)Pr(h)/Pr(r) = (3/10)/(13/20) = **6/13**

The biggest difficulty with this problem, if you go directly to the equations, is identifying the statements whose probability matters. The things that matter are whether the tank is half full and whether the gauge reads ½ full. Less important is whether the gauge is accurate. That will matter for computing the probability that the gauge reads ½ full, but it will not matter otherwise. Start your search with a careful statement of the claim you are trying to determine, in this case: the probability that the tank is half full, given that the gauge reads ½.

The gas gauge on your new car always works well. One of your friends borrowed it last night without asking, but you're not sure whether it was The Dude or Maude. 70% of the time, it was the Dude, while 30% of the time, it was Maude. As you know, 70% of the time, The Dude returns your car with an almost empty tank, while 30% of the time, he returns it half full. Maude, more virtuous, returns it half full 70% of the time and empty only 30% of the time. You get in your car and see the gauge indicating ½ tank. In these circumstances, what is the probability that Maude borrowed the car? Show your work and clearly indicate your answer.

pr(M | h) = [pr(h | M) * pr(M)] / pr(h) = 7/10*3/10 / pr(h) = **1/2**

> pr(h) = pr[(h & M) v (h & -M)] =
> pr(h | M)pr(M) + pr(h | -M)pr(-M) =
> 7/10*3/10 + 3/10*7/10 = 42/100

Back of the envelope:

You get the car returned 100 times. 70 of those cases were The Dude, 30 were Maude. Among The Dude's 70, 49 were returned half empty, and 21 were half full. Among Maude's 30, 21 were half full, 9 were empty. So, out of 42 total cases in which the car was returned half full, 21, or half of them were returned by Maude.

One chart to rule them all! The chart is easy, because you know the unconditioned probabilities that The Dude, or Maude, return the car (rightmost column), then filling in is easy in light of the claims about how often each returns the car ½ full or empty. And both questions are readily answered in light of it.

	½ tank	Empty	
Dude	21/100	49/100	70/100
Maude	21/100	9/100	30/100
	42/100	58/100	1

Let's say we have two bags, each of which contains a bunch of Scrabble tiles (you know: little wooden things, each of which has a single letter on it). The tiles in one bag spell out 'battleship', while the tiles in the other spell out 'bayonet'. You are looking for the 'bayonet' bag. If you choose a bag at random, the probability that you get the 'bayonet' bag is obviously ½. (Tricky!)

What is the probability that you have the 'bayonet' bag if you draw a 'b' from it?

Pr(bayonet bag|draw b) =
Pr(draw|bayonet)Pr(bayonet)/Pr(draw b)= (1/7)(1/2)/Pr(draw b)

Pr(draw b) = Pr[(draw b & bayonet) v (draw b & not bayonet)] = (1/2)(1/7) + (1/2)(1/10)
Note: This value is almost equal to 2/17, it's not quite that because there are different numbers of tiles in each bag. This fact makes these problems trickier than you might have thought.

Pr(bayonet bag|draw b) = (1/7)/(1/7+1/10) = 10/17

What is the probability that you have the 'bayonet' bag if you draw a 't' from it?

Pr(bayonet|draw t) = Pr(draw t|bayonet)Pr(bayonet)/Pr(draw t) = (1/7)(1/2)/Pr(draw t)
Pr(draw t) = Pr[(draw t & bayonet) v (draw t & not bayonet)] = (1/2)(1/7) + (1/2)(2/10)
Pr(bayonet|draw t) = (1/7)/(1/7 + 2/10) = 10/24

What is the probability that you have the bayonet bag if you draw a 't' from each bag?
Pr(bayonet|draw tt) =
Pr(draw tt|bayonet)Pr(bayonet)/Pr(draw tt)
Notice, in this case, that
Pr(draw tt|bayonet) = Pr(draw tt|not bayonet) = Pr(draw tt),
so we can simplify:
Pr(bayonet|draw tt) = Pr(bayonet) = ½

Jack and Jill pack their lunch for a hike up the hill. Jack's bag of sandwiches: two cheese, one tuna. Jill's: two tuna, one cheese. They sit down and each pulls one bag from the backpack, but they're not yet sure whose is whose. They reach in to their respective bags and withdraw one sandwich each. What is the probability that they have the correct bags (i.e. Jack has Jack's bag and Jill has Jill's) if…

*There are **four** ways to do this. Let's check them all.*

…Jack draws a tuna and Jill draws a cheese?

1/5: There are five equally probable ways to get this combo. In one of them, Jack has his bag and she has hers. In the other four, Jack has one or another of hers while she has his.

Jack has:	*Jill has:*
Jack's bag	Jill's bag
Jill's 1st tuna	Jack's 1st cheese
Jill's 2nd tuna	Jack's 1st cheese
Jill's 1st tuna	Jack's 2nd cheese
Jill's 2nd tuna	Jack's 2nd cheese

…Jack draws a tuna and Jill draws a tuna?

1/2: There are four equally probable ways to get this combo.

Jack has:	*Jill has:*
Jill's bag	Jack's 1st tuna
Jill's bag	Jack's 2nd tuna
Jack's 1st tuna	Jill's bag
Jack's 2nd tuna	Jill's bag

…Jack draws a cheese and Jill draws a tuna?

4/5: Intuitive enough, given the first answer:

Jack has:	*Jill has:*
Jill's bag	Jack's bag
Jack's 1st cheese	Jill's 1st tuna
Jack's 1st cheese	Jill's 2nd tuna
Jack's 2nd cheese	Jill's 1st tuna
Jack's 2nd cheese	Jill's 2nd tuna

…Jack draws a cheese and Jill draws a cheese?

1/2: Same reasoning as the second response.

Here's the second way of doing it:

…Jack draws a tuna and Jill draws a cheese?

If they have the right bags, the probability of Jack drawing tuna and Jill drawing cheese is:
Pr(t|right bag)Pr(c|right bag) = 1/3*1/3 = 1/9

But the probability of t&c if they have the wrong bags: Pr(t|wrong bag)Pr(c|wrong bag)=2/3*2/3 = 4/9

So, in 1 out of five of the cases in which Jack draws tuna and Jill draws cheese, they have the right bags.

Common mistake:
Some just gave 1/9 and 4/9 as their answers to the first and third questions, along with 2/9 for the second and fourth. You got part of the way to the answer. These give the probabilities that such combinations are drawn, but not the probability that they have the right bags, given that such combinations are drawn. I took off half the points, or 6 out of 12, for this mistake.

…Jack draws a tuna and Jill draws a tuna?

If they have the right bags,
Pr(t|right)Pr(t|right) = 2/3*1/3 = Pr(t|wrong)Pr(t|wrong)
So the odds are even that they have the right/wrong bags and the answer is 1/2.

…Jill draws a tuna and Jack draws a cheese?

You see how this works…

…Jack draws a cheese and Jill draws a cheese?
Here too.

Third, brute force method:

…Jack draws a tuna and Jill draws a cheese?

Pr(They have the right bags | Jack draws tuna and Jill cheese) =
Pr(Jack tuna&Jill cheese|Right)Pr(Right)/P(Jack tuna&Jill cheese)
=[(1/3*1/3)* ½]/ Pr(Jack tuna & Jill cheese)

Pr(Jack tuna & Jill cheese) =
Pr[(Jack tuna&Jill cheese&right)v(Jack tuna&Jill cheese& wrong)]
= Pr(Jack tuna&Jill cheese|Right)Pr(Right) +
Pr(Jack tuna&Jill cheese|wrong)Pr(wrong)
=(1/3*1/3)*½ + (2/3*2/3)*½ = ½ (1/9 + 4/9) = ½*5/9

So, substituting back we get:
[(1/3*1/3)* ½]/ Pr(Jack tuna & Jill cheese) =(1/9*½)/(½*5/9) = **1/5**

…Jack draws a tuna and Jill draws a tuna?

Pr(They have the right bags | Jack draws tuna and Jill tuna) =
Pr(Jack tuna&Jill tuna|Right)Pr(Right)/Pr(Jack tuna&Jill tuna) =
(1/3*2/3)½/Pr(Jack tuna & Jill tuna)

Pr(Jack tuna & Jill tuna) =
Pr[(Jack tuna&Jill tuna&Right)v(Jack tuna&Jill tuna&Wrong)] =
Pr(Jack tuna & Jill tuna & Right)Pr(Right) +
Pr(Jack tuna & Jill tuna & wrong)Pr(Wrong) =
(1/3*2/3)½ + (1/3*2/3)½ = (1/3*2/3) = 2/9

So, substituting back we get:

(1/3*2/3)½/Pr(Jack tuna & Jill tuna) = (1/3*2/3)½/ (1/3*2/3) = **½**

The same reasoning gets you answers to the other two questions by this method.

And, fourth, as a table:

We need to choose between the situation in which they have the right bags and the situation in which they have the wrong bags. The tests (drawing the sandwiches) can some up in four ways: they both get cheese, or tuna, or they get different sandwiches.

The top row indicates Jack's draw and then Jill's, so 'ct' means: Jack draws cheese and Jill draws tuna.

	cc	tt	tc	ct	
Right bags	1/9	1/9	1/18	4/18	½
Wrong bags	1/9	1/9	4/18	1/18	½
	2/9	2/9	5/18	5/18	1

So, when Jack draws tuna and Jill cheese (third column), in 1 out of five cases they have the right bags, and in 4 out of five they have the wrong ones.

When they draw the same kind of sandwich (tuna or cheese), they have even odds of having the right bags.

When Jack draws cheese and Jill tuna, in four out of five cases they have the right bags, and in the other case they have the wrong ones.

We want to know who has the disease. We have two tests, described below. Let's use 'd' to represent the claim that someone has the disease, and 't' to represent the claim that someone tests positive for the disease.

The base rate of the disease is 1%:		pr(d) = 1/100
Test 1:	Sensitivity 70%:	pr(t | d) = 7/10
	Selectivity 100%:	pr(t | -d) = 0
Test 2:	Sensitivity 100%:	pr(t | d) = 1
	Selectivity 70%:	pr(t | -d) = 3/10
	Note: 70% selectivity means 30% false positive rate.	

For each test, calculate pr(d | t).

Test 1: pr(d | t) = pr(t | d)pr(d)/pr(t) = (7/10*1/100)/pr(t)

pr(t) = pr(d)pr(t | d) + pr(-d)pr(t | -d) = 1/100*7/10 + 99/100*0 = 7/1000

So, pr(d | t) = (7/1000)/(7/1000) = 1

Or, commonsensically, since there are no false positives, everyone who tests positive has the disease.

Test 2: pr(d | t) = pr(t | d)pr(d)/pr(t) = (1/100)/pr(t)

pr(t) = pr(d)pr(t | d) + pr(-d)pr(t | -d) = 1/100 + 99/100*3/10
= 1/100 + 297/1000 = 307/1000

So, pr(d | t) =(1/100)/(307/1000) = 10/307

If we test 1000 people, the 10 who have the disease test positive, and around 300 (indeed 297) who are healthy also test positive.

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Consider the following testing strategies:

A: First test everyone with Test 1. Then apply Test 2 only to those who tested positive to Test 1.

B: First test everyone with Test 2. Then apply Test 1 only to those who tested positive to Test 2.

C: First test everyone with Test 2. Then apply Test 2 again only to those who tested positive the first time.

For each strategy, calculate the probability that a person has the disease, given that she has tested positive in both rounds of testing. *5 points each*

A: Test 1 yields no false positives, so the new base rate is 100%. Since everyone subsequently taking test 2 has the disease, everyone who tests positive for it with test 2 has the disease.

B: Not everyone who tests positive for test 2 actually has the disease, but you learned that everyone who takes test one and tests positive does have the disease, so everyone who tests positive for both has the disease.

C: In this case, treat the results of the first test 2 as changing the base rate in the population you test. The new base rate is 10/307.

Turn the crank:
pr(d | t) = pr(t | d)pr(d)/pr(t) = (10/307)/pr(t)
pr(t) = pr(d)pr(t | d) + pr(-d)pr(t | -d) = 10/307 + 297/307*3/10
= (100 + 891)/3070 = 991/3070
So, pr(d | t) = (10/307)/(991/3070) = 100/991

Back of the envelope:
Imagine we have 307 people, 10 of whom have the disease. The second application of test 2 captures all 10 who have it and 30% of the remaining 297. We know that 297*3/10 = 891/10 = 89.1. Overall, we have 10 + 89.1 = 99.1 positive tests, and 10 of those folks actually have the disease, so pr(d | t) = 10/99.1 = 100/991.

For which strategy does the greatest number of sick people test positive in both rounds? *5 points*

This should be easy: C. Test 2 doesn't miss any sick people. It does suggest a lot of healthy people are sick as well, but it gets as many sick people as any test could: all of them. Strategies A and B make use of test 1, which misses some sick people.

If it were essential to isolate all and only people with the disease, how could you do that with repeated applications of these two tests? Explain. *5 points*

Test 2 gets all the sick people, while test 1 gets only sick people, though it misses some.

The surest path is as follows: Apply test 2, then apply test 1. Those who test positive have the disease. Remove those who test positive to both from the pool, then apply test 2 again, followed by 1, and so on. It's possible that after a round of two negative tests that there are some people left with the disease, but you can be surest the quickest on this path.

Another way is to apply test 1, remove those who test positive, then apply test 1 again only to those who tested negative, remove and repeat. This process will be a slog, and you will never be sure you're done, as Test 1 might keep missing some sick people. I took off one point for this.

A chancy method would be to apply test 2, and then apply it again only to those who test positive. Eventually, you should narrow down the field to all and only sick people. The worry with this strategy is that there will always be some chance that you have classified a healthy person as sick. I took off one point for this proposal.

The philosophy and religion departments are gearing up for their annual holiday raffles. Each raffle has 100 tickets and they both always sell out. For each raffle, the tickets cost the same, and the prizes are the same. Each raffle has exactly one winner and the winner is the one who buys the winning ticket. You can afford to buy four tickets. Which strategy, if any maximizes the probability that you win at least one raffle?

1. Buy two tickets in each raffle.
2. Buy four tickets in one raffle.
3. Buy 3 in one and 1 in the other.

Or is it true that

4. All of the previous strategies are equally likely to produce a win.
5. The first two are equally likely to produce a win.

Answer: 2. Buy four tickets in one raffle.

What is the probability that:

you win the philosophy raffle if you buy four tickets? 4/100

you win the philosophy raffle if you buy two tickets? 2/100

you win the philosophy raffle if you buy one ticket? 1/100

you win philosophy or religion if you buy two tickets in each?
Pr(win phil v win rel) =
Pr(win phil) + Pr(win rel) – Pr(win phil & win rel) =
1/50 + 1/50 – 1/2500 = 99/2500 = .0396
(just a bit worse than 4/100, or 1/25)

Or, you could think of it as the probability that you win at least once, in two tries at identical lotteries:
Pr (win once in two tries) = $1 - [\Pr(\text{-win})]^2 = 1 - (49/50)^2 =$
1 - .9604 = .0396

you win philosophy or religion if you buy one phil and 3 rel?
Pr(win phil v win rel) =
Pr(win phil) + Pr(win rel) – Pr(win phil & win rel) =
= 1/100 + 3/100 – 3/10000 = 397/10000
(also a bit worse than 4/100)

you win philosophy *and* religion if you buy two in each?
Pr(win phil & win rel) = Pr(win phil)Pr(win rel) =
1/50*1/50 = 1/2500

you win philosophy and religion if you buy one phil and 3 rel?
Pr(win phil & win rel) = Pr(win phil)Pr(win rel) =
1/100*3/100 = 3/10000

you win philosophy and religion if you only buy 4 phil tickets?
0 (Duh.)

Which of the three strategies above (1, 2, or 3) has the greatest expected monetary value?
The cost of playing is the same and the only difference in payoffs is that were one to win both one would win twice the money. So, we don't need a specific payout or ticket price to determine which strategy has the greatest expected value. Take X to be the payout.

1. 2X/2500 + 98X/2500 = 100X/2500 = X/25; (In 1/2500 cases you win both (2X payout) and in the other 98 you win only X)
2. X/25; You buy 4 tickets, so your probability of winning X is 4/100 or 1/25.
3. 6X/10000 + 394X/10000 = 400X/10000 = X/25 (In 3 out of 10000 cases you win 2X and in the other 394 cases you win X.)

There is no expression for how much you lose here because in each case that number is the same (P(24/25), where P is the price of a ticket) and all the question asks is which strategy has the greatest expected monetary value. They are, as you can see, equivalent. Why?

Game night in the Philosophy Department was cancelled a couple of years ago, after Professor Levey, out of money, tried to bet his dog on a game of Woozy. It's a very fun game. Without looking, you pick two cards from a deck. If they are consecutive, you win, if not, you lose, and everybody yells "Woozy!". It costs $40 to play, but you are given $250 if you win. Just in case you don't know, a standard deck includes 52 cards: four suits—hearts, diamonds, clubs, and spades—each of which includes 13 cards—A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K. Consecutive cards are next to one another in order, so, 4 and 5, or 4 and 3 are consecutive. King and Ace or King and Queen are consecutive. Ace and 2, or 2 and 3 are consecutive, and so on. Suits don't matter for consecutiveness, so Ace of spades and King of hearts are consecutive. Identical cards—4 and 4, e.g.—are not consecutive.

What it the probability of winning Woozy?

Easy: you draw a card, and want to know the probability that the other one is consecutive with it. There are eight cards consecutive with any other card: four suits for the card above and the card below the one in question. Since you already drew one, those eight must come from 51, not 52, so the probability that you win Woozy is 8/51.

What is the expected monetary value of playing Woozy?

EMV(play Woozy) = pr(win)MV(win) + pr(lose)MV(lose)
= 8/51*210 + 43/51*-40 = (1680 – 1720)/51 = -40/51

Remember to subtract 40 from 250. You are given 250 if you win, but you paid 40 to play, so the winnings are 210.

Is it a good idea, financially speaking, to play Woozy? Explain.

No. A negative expected monetary value indicates that if you play Woozy, overall you can expect to lose money. Sam tried to bet his dog because he lost all his money. Learn from his fail. If you just said that you should not play because EMV is less than zero, I took off two points. Explain your answer.

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If you win, you are offered the chance to "go WoozyWoozy". You bet the $250 you just made, and pick a third card from the deck. If that card is consecutive with the other two you drew, you win $1500. Otherwise, you lose your bet.

Assume you have already won Woozy. What is the probability of winning WoozyWoozy?

This is easy too. We have 50 cards left in the deck, 8 of which are consecutive with the two cards you have (four above, four below). So, the probability of winning is 8/50.

What is the expected monetary value of playing WoozyWoozy? *5 points*

1250*8/50 + -250*42/50 = 10000/50 – 10500/50 = -500/50 = -10

Is it a good idea, financially speaking, to go WoozyWoozy? Explain.

No. Same reason as above.

Would you go WoozyWoozy?

!